M.SC. FOURTH SEMESTER EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 42153**

Course Code: Math-403ME

Course Title: Modelling and Analysis of Biological systems (Old)

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

Answer any five from the following questions. $8 \times 5 = 40$

1. The population of India in 1951 and 2021 are 38.23 crore and 139.34 crore, respectively.
   a) Considering that the human population grows according to exponential growth, find the growth rate (r) of human population in India. Also estimate the population size in India for the year 2030.
   b) If human population in India grows logistically with intrinsic growth rate $r = 0.95$ per year and carrying capacity 250 crores. Now considering logistic growth and 2021 as the initial time point estimate the population size in India for the year 2050. $(3 + 2) + 3 = 8$

2. Write down the spruce budworm insect outbreak model. Using graphical method investigate the stability property of the equilibrium points. Also discuss the occurrence of saddle node bifurcation for gradual increase in the growth rate of insect population. $2 + 3 + 3 = 8$

3. The Rosenzweig-MacArthur model is given as

$$\frac{dN}{dT} = rN\left(1 - \frac{N}{K}\right) - \frac{cNP}{a + N}$$
$$\frac{dP}{dT} = \frac{bNP}{a + N} - mP.$$

   a) Considering the dimensionless variables $x = \frac{N}{K}$, $y = \frac{P}{K}$, $t = rT$, derive the dimensionless model of the Rosenzweig-MacArthur model.
   b) Find the equilibrium points and investigate their local stability for the above dimensionless model.
   c) Also discuss the occurrence of Hopf bifurcation around the interior equilibrium.

$2 + 4 + 2 = 8$

4. a) Write down the Leslie-Gower model. Find the steady states and investigate the stability of the interior equilibrium point.

b) Write down the major differences between Leslie-Gower model and Rosenzweig-MacArthur model. $(1+4)+3=8$

5. a) Write down the modified Lotka-Volterra model.

b) Using Bendixson-Dulac's criteria show that the above system has no close orbit.

c) Also investigate the global stability of the interior equilibrium. $2+3+3=8$

6. (a) Write down the biological assumptions and sketch the schematic diagram for the SIR (susceptible-infected-recovery) epidemic outbreak. Formulate the SIR model.

(b) Find the Basic reproduction number for the disease based on the above model. Also investigate the global stability of the disease-free equilibrium point. $(2+1+1)+(2+2)=8$

7. (a) Write down the biological assumptions and formulate a three species intraguild predation model.

(b) Find out the condition(s) for stability of intraguild predator (top predator) free equilibrium point. $4+4=8$

8. (a) Write down the biological assumptions, draw the schematic diagram, and formulate an SEIR model for capturing the long-term behavior of Covid 19 outbreak.

(b) Investigate the local stability of the disease-free equilibrium point in terms of basic reproduction number (R_0). $4+4=8$

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